

Are You Ready for Key Calculus?

Summer Math Review Packet



①

This packet contains math problems, formulas, concepts, and examples that will be useful to you in Calculus. It is important that you work the problems, check your answers, and study the concepts presented. Most, if not all, of this packet should be familiar to you. Work all of the problems on your own paper and have this ready to turn in to your math teacher on the first day you return to school in September. Show all work neatly and orderly.

Factor each expression completely.

1. $x^3 + 8$

$$(x)^3 + (2)^3$$

$$(x+2)(x^2 - 2x + 4)$$

2. $x^3 - 8$

$$(x)^3 - (2)^3$$

$$(x-2)(x^2 + 2x + 4)$$

3. $x^2 + 11x - 80$

$$ac = -80$$

$$\begin{matrix} -5 & 16 \end{matrix}$$

$$(x-5)(x+16)$$

4. $ac + cd - ab - bd$

$$c(a+d) - b(a+d)$$

$$(c-b)(a+d)$$

5. $2x^2 + 50y^2 - 20xy$

$$2(x^2 + 25y^2 - 10xy)$$

$$2(x^2 - 10xy + 25y^2)$$

$$\begin{matrix} 25 \\ -5 & -5 \end{matrix}$$

$$2(x-5y)(x-5y) = 2(x-5y)^2$$

6. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

$$(x-3)^2(2x+1)^2 [(2x+1) + (x-3)]$$

7. $6x^5 - 51x^3 - 27x$

$$3x(2x^4 - 17x^2 - 9)$$

$$2 \cdot -9 = -18$$

$$\begin{matrix} -18 & 1 \end{matrix}$$

$2x^2$	$x^2 - 9$
$2x^4$	$-18x^2$
$1x^2$	-9

Box method

$$3x(2x^2 + 1)(x^2 - 9)$$

8. $3x^3 - 2x^2 - 12x + 8$

$$x^2(3x-2) - 4(3x-2)$$

$$(x^2 - 4)(3x-2)$$

$$(x-2)(x+2)(3x-2)$$

Simplifying Expressions

Simplify each expression. Write answers with positive exponents where applicable

$$9. (4a^{\frac{5}{3}})^{\frac{3}{2}}$$

$$\frac{4^{\frac{3}{2}} a^{\frac{5}{2}}}{4^{\frac{3}{2}} a^{\frac{5}{2}}} = \boxed{8 a^{\frac{5}{2}}}$$

$$10. \frac{\frac{2}{x^2} \cdot \frac{x^3}{10}}{\frac{10}{x^3} \cdot \frac{x^3}{10}}$$

$$\frac{2x^3}{10x^2} = \frac{x}{5} = \boxed{\frac{1}{5}x}$$

$$11. \left(\frac{\frac{1}{2} \cdot \frac{5}{4}}{\frac{3}{8}} \right)^{\frac{8}{3}}$$

$$\frac{8}{3} \left(\frac{1}{2} - \frac{5}{4} \right)$$

$$\frac{8}{6} - \frac{40}{12} = \frac{16}{12} - \frac{40}{12} = \frac{-24}{12} = \boxed{-2}$$

$$12. \frac{12x^{-3}y^2}{18xy^{-1}}$$

$$\frac{12y^2y}{18x^4x^3} = \frac{12y^3}{18x^4} = \boxed{\frac{2y^3}{3x^4}}$$

$$13. \frac{5-x}{x^2-25}$$

$$\frac{5-x}{(x-5)(x+5)} = \frac{-1}{-1}$$

$$\frac{x-5}{(x-5)(-x-5)} = \boxed{\frac{1}{-x-5}}$$

$$14. \frac{15x^2}{5\sqrt{x}} = \frac{15x^2}{5x^{1/2}} = \boxed{3x^{3/2}}$$

Complex Fractions

Simplify each of the following expressions.

$$15. \frac{\frac{25}{a} - a}{5+a} \cdot \frac{a}{a}$$

$$\frac{25 - a^2}{5a + a^2}$$

$$\frac{(5-a)(5+a)}{a(5+a)} = \boxed{\frac{5-a}{a}}$$

$$16. \frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}} \cdot \frac{(x+2)}{(x+2)}$$

$$\frac{2(x+2) - 4}{5(x+2) + 10} = \frac{2x + 4 - 4}{5x + 10 + 10} = \frac{2x}{5x + 20} = \boxed{\frac{2x}{5(x+4)}}$$

$$17. \frac{\frac{4}{x^2-9} + \frac{2}{x-3}}{\frac{1}{x+3} + \frac{1}{x-3}}$$

$$\frac{\frac{4}{(x-3)(x+3)} + \frac{2}{(x-3)}}{\frac{1}{x+3} + \frac{1}{x-3}} \cdot \frac{(x+3)(x-3)}{(x+3)(x-3)}$$

$$\frac{4 + 2(x+3)}{(x-3) + (x+3)} = \frac{4 + 2x + 6}{2x + 6} = \frac{2x + 10}{2x + 6} = \frac{2(x+5)}{2(x+3)} = \frac{x+5}{x+3}$$

Solving Rational Equations
Solve each equation for x.

$$19. \frac{2}{3} - \frac{5}{6} = \frac{1}{x}$$

$$\frac{4}{6} - \frac{5}{6} = \frac{-1}{6}$$

$$\boxed{x = -6}$$

$$\boxed{\frac{x+5}{x}}$$

$$18. \frac{\frac{36}{1+\frac{7}{x}} \cdot \frac{x}{x}}{2+7}$$

$$\frac{\frac{36x}{1+\frac{7}{x}} \cdot \frac{2}{2}}{2+7} = \frac{72x}{9} = \boxed{8x}$$

$$20. \frac{x-5}{x+1} = \frac{3}{5}$$

$$5(x-5) = 3(x+1)$$

$$5x - 25 = 3x + 3$$

$$\begin{array}{r} -3x \\ \hline 2x - 25 = 3 \end{array}$$

$$\frac{2x - 25 = 3}{25 \quad 25}$$

$$\frac{2x}{2} = \frac{28}{2}$$

$$22. \frac{x}{2x-6} - \frac{3}{x^2-6x+9} = \frac{x-2}{3x-9}$$

$$\boxed{x = 14}$$

$$21. \frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$$

$$\frac{2(x-5)}{(x+5)(x-5)} + \frac{(x+5)}{(x-5)(x+5)} = \frac{16}{x^2-25}$$

$$\frac{2(x-5) + (x+5)}{(x-5)(x+5)} = \frac{16}{x^2-25}$$

$$\frac{2x - 10 + x + 5}{x^2 - 25} = \frac{16}{x^2 - 25}$$

$$\frac{3x - 5}{x^2 - 25} = \frac{16}{x^2 - 25}$$

$$16 \left(\frac{x^2-25}{x^2-25} \right) = \frac{(3x-5)(x^2-25)}{(x^2-25)}$$

$$16 = 3x - 5$$

$$21 = 3x$$

$$\boxed{7 = x}$$

Please see next page!

Functions

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each of the following:

23. $f(2) = 2(2) + 1 = 5$

~~$f(x)$~~

25. $f(t+1) = 2(t+1) + 1 = 2t + 2 + 1 = 2t + 3$

27. $g(f(m+2)) = f(m+2) = 2(m+2) + 1 = 2m + 4 + 1 = 2m + 5$

$g(2m+5) = 2(2m+5)^2 - 1$

$2(2m+5)(2m+5) - 1 = 2[4m^2 + 10m + 10m + 25] - 1 = 8m^2 + 40m + 49$

Let $f(x) = x^2$, $g(x) = 2x + 5$ and $h(x) = x^2 - 1$. Find each of the following

29. $f(g(x-1)) = g(x-1) = 2(x-1) + 5 = 2x - 2 + 5 = 2x + 3$

$F(2x+3) = (2x+3)^2 = (2x+3)(2x+3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$

24. $g(-3) = 2(-3)^2 - 1 = 2(9) - 1 = 18 - 1 = 17$

26. $f(g(-2)) = g(-2) = 2(-2)^2 - 1 = 2(4) - 1 = 8 - 1 = 7$
 $f(7) = 2(7) + 1 = 15$

28. $\frac{f(x+h) - f(x)}{h} = \frac{2(x+h) + 1 - (2x + 1)}{h} = \frac{2x + 2h + 1 - 2x - 1}{h} = \frac{2h}{h} = 2$

$2x + 2h + 1 - [2x + 1] = 2h$

30. $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h$

$2x + h$

22.

$$\frac{x}{2x-6} - \frac{3}{x^2-6x+9} = \frac{x-2}{3x-9}$$

2
3
(x-3)(x-3)

$$\frac{x}{2(x-3)} - \frac{3}{(x-3)(x-3)} = \frac{x-2}{3(x-3)}$$

$$\frac{x(x-3)(x-3)}{2(x-3)(x-3)(x-3)} - \frac{6(x-3)}{2(x-3)(x-3)(x-3)} = \frac{x-2}{3(x-3)}$$

$$\frac{x(x-3)(x-3) - 6(x-3)}{2(x-3)(x-3)(x-3)} = \frac{x-2}{3(x-3)}$$

$$\frac{x(x-3) - 6}{2(x-3)(x-3)} = \frac{x-2}{3(x-3)}$$

$$3x(x-3)(x-3) - 18(x-3) = 2(x-3)(x-3)(x-2)$$

$$3x(x-3) - 18 = 2(x-3)(x-2)$$

$$3x^2 - 9x - 18 = (2x-6)(x-2)$$

$$3x^2 - 9x - 18 = 2x^2 - 4x - 6x + 12$$

$$3x^2 - 9x - 18 = 2x^2 - 10x + 12$$

$$-2x^2$$

$$-2x^2$$

$$x^2 - 9x - 18 = -10x + 12$$

$$x^2 + x - 18 = 12$$

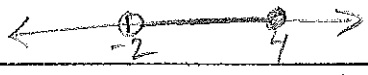


$$x^2 + x = 30 \rightarrow (x+6)(x-5) = 0$$

$$x^2 + x - 30 = 0 \rightarrow x = -6, 5$$

Interval Notation

Complete the table with the appropriate notation or graph

31. Complete the table with the appropriate notation or graph

Solution	Interval Notation	Graph
$-2 < x \leq 4$	$(-2, 4]$	
$-1 \leq x < 7$	$[-1, 7)$	
$x \leq 8$	$(-\infty, 8]$	

Domain and Range

Find the domain and range of the following functions.

32. $f(x) = x^2 - 5$

D: \mathbb{R}

R: $y \geq -5$

33. $f(x) = -\sqrt{x+3}$

D: $x \geq -3$

R: $y \leq 0$

34. $f(x) = 3\sin x$

D: \mathbb{R}

R: $[-3, 3]$

35. $f(x) = \frac{2}{x-1}$

D: $(-\infty, 1) \cup (1, +\infty)$

R: $(-\infty, 0) \cup (0, +\infty)$

Inverses

Find the inverse for each function.

36. $f(x) = 2x + 1$

$x = 2y + 1$

$\frac{x-1}{2} = \frac{2y}{2}$

$\frac{x-1}{2} = y$

37. $f(x) = \frac{x^2}{3}$

$x = \frac{y^2}{3}$

$3x = y^2$

$\pm\sqrt{3x} = y$

6

What is the relationship of inverses.....

Numerically? $(x,y) \rightarrow (y,x)$ or $F(y) = x$

Graphically? reflection over line $y = x$

Algebraically? $F(g(x)) = g(F(x)) = x$

Prove f and g are inverses of each other.

38. $f(x) = 9 - x^2, x \geq 0$ and $g(x) = \sqrt{9 - x}$

$$\begin{aligned}
 f(g(x)) &= 9 - (\sqrt{9-x})^2 \\
 &= 9 - 9 + x \\
 &= x
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= \sqrt{9 - (9 - x^2)} \\
 g(f(x)) &= \sqrt{-x^2} \\
 g(f(x)) &= (-x^2)^{1/2} \\
 g(f(x)) &= -x
 \end{aligned}$$

*Composites are equal

Equations of Lines

Slope Intercept Form: $y = mx + b$

Point Slope Form: $y - y_1 = m(x - x_1)$

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

$X = 5$

Vertical line = undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

$$\begin{aligned}
 y - 2 &= 0(x - -4) \\
 y - 2 &= 0(x + 4)
 \end{aligned}$$

$$\begin{aligned}
 y - 2 &= 0x + 0 \\
 y &= 2
 \end{aligned}$$

horizontal line

41. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

parallel $\rightarrow m = \frac{5}{6}$

$$y - 8 = \frac{5}{6}(x - 2)$$

$$\begin{aligned}
 y - 8 &= \frac{5}{6}x - \frac{10}{6} \\
 y - 8 &= \frac{5}{6}x - \frac{5}{3}
 \end{aligned}$$

$$y = \frac{5}{6}x - \frac{19}{3}$$

42. Find the equation of a line perpendicular to the y-axis passing through the point (4, 7).

y-axis $\rightarrow 0$ slope

perpendicular line $\rightarrow m = \frac{1}{0}$ or undefined so vertical line.

$X = 4$

43. Find the equation of a line passing through the points (-3, 6) and (1, 2).

$$m = \frac{2-6}{1-(-3)} = \frac{-4}{4} = -1$$

$$\begin{aligned}
 y - 2 &= -1(x - 1) \\
 y - 2 &= -x + 1
 \end{aligned}$$

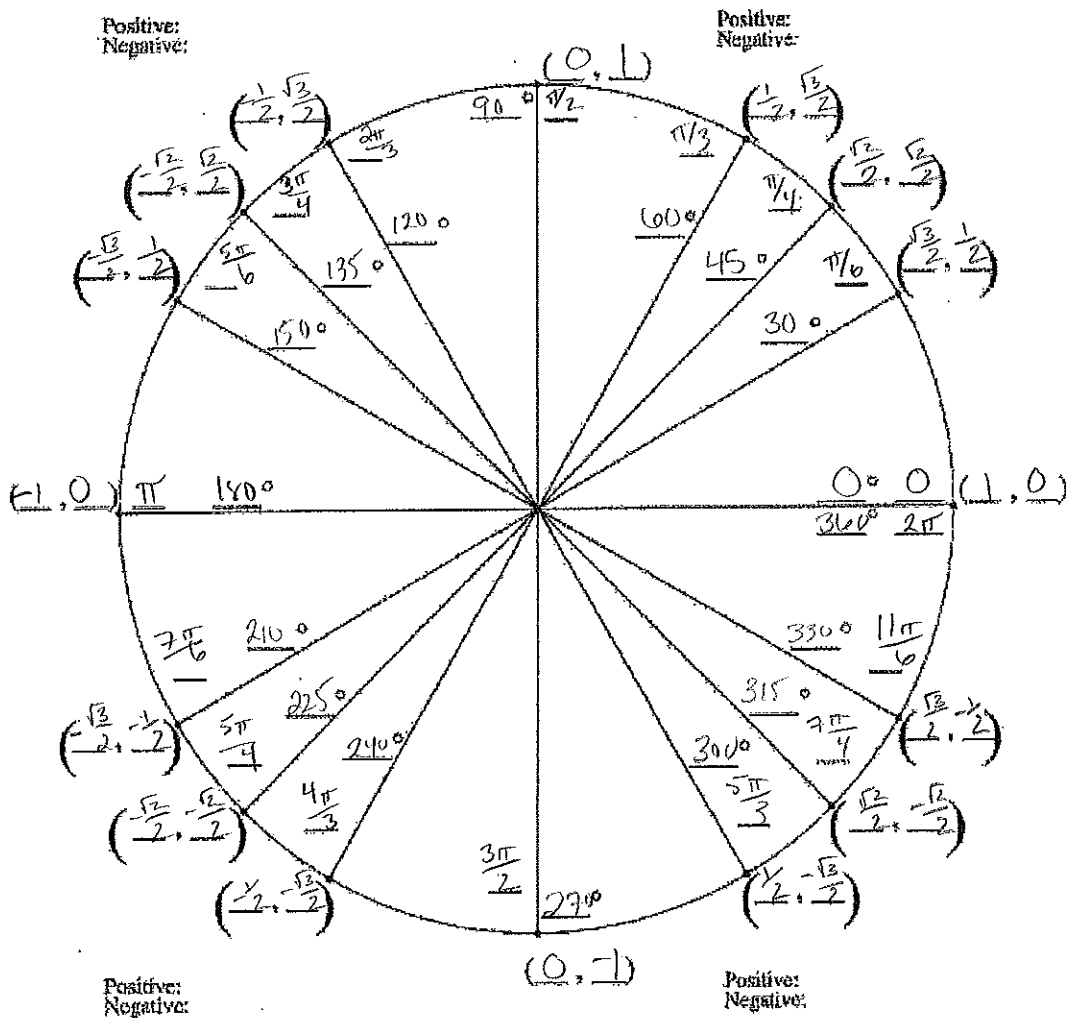
$y = -x + 3$

44. Find the equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

$$m = \frac{3-0}{0-2} = \frac{3}{-2}$$

$y = -\frac{3}{2}x + 3$

Fill in The Unit Circle



Evaluate...exactly...no calculator

45. $\sin \pi = 0$

47. $\sin(-\frac{\pi}{2}) = -1$

49. $\tan \frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

51. $\tan \frac{\pi}{2} = \text{undefined} \rightarrow \frac{1}{0}$

46. $\cos \frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$

48. $\sin \frac{11\pi}{6} = \frac{-1}{2}$

50. $\sec \frac{7\pi}{6} = \frac{1}{\cos \frac{7\pi}{6}} = \frac{1}{\frac{-\sqrt{3}}{2}} = \frac{2}{-\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

52. $\csc \frac{\pi}{3} = \frac{1}{\sin \frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

Logarithms

Fill in the following properties for logarithms:

$$\begin{aligned} \log_b 1 &= \underline{0} \\ \log_b b &= \underline{1} \\ \log_b b^x &= \underline{x} \\ \log_b uv &= \underline{\log_b u + \log_b v} \\ \log_b \frac{u}{v} &= \underline{\log_b u - \log_b v} \\ \log_b u^n &= \underline{n \log_b u} \end{aligned}$$

Evaluate without a calculator

53. $\log_4 64 = \boxed{3}$

$4^x = 64$

$x = 3$

55. $\ln e^8 = \boxed{8}$

54. $\log_8 \frac{1}{2} = \boxed{-\frac{1}{3}}$

$8^x = \frac{1}{2}$

$\frac{x \log 8}{\log 8} = \frac{\log \frac{1}{2}}{\log 8} \Rightarrow x = -\frac{1}{3}$

56. $\log_{1/3} 27 = \boxed{-3}$

$\frac{1}{3}^x = 27$

$\frac{x \log \frac{1}{3}}{\log \frac{1}{3}} = \frac{\log 27}{\log \frac{1}{3}} \Rightarrow x = -3$

Solve. You may use a calculator for the last step, although exact answers (no decimals) will be accepted.

57. $12 = 10^{x+5} - 7$
 $\frac{+7}{+7}$
 $19 = 10^{x+5}$

$\frac{\log 19}{\log 10} = \frac{(x+5) \log 10}{\log 10}$

$\frac{\log 19}{\log 10} - 5 = x$

59. $3e^{-x} - 4 = 9$
 $\frac{+4}{+4}$
 $3e^{-x} = 13$

$\frac{3}{3} e^{-x} = \frac{13}{3}$

$e^{-x} = \frac{13}{3}$

$\ln e^{-x} = \ln \frac{13}{3}$

$-x = \ln \frac{13}{3}$

$x = -\ln \frac{13}{3}$

58. $5 - \ln x = 7$
 $\frac{-5}{-5}$
 $-\ln x = 2$

$\ln x = -2$
 $e^{\ln x} = e^{-2}$

$x = e^{-2}$

60. $5 \log_3(x-2) = 10$
 $\frac{5}{5}$
 $\log_3(x-2) = 2$

$3^2 = x-2$

$9 = x-2$

$9 + 2 = x$

$11 = x$